

New Energy-Momentum and Angular Momentum Tensors with Applications to Nucleon Structure

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Abstract

We present a new type of energy-momentum tensor and angular momentum tensor. They are motivated by a special consideration in quantum measurement: Given a wave in mutual eigen-state of more than one physical observables, the corresponding physical currents should be proportional to each other. Interestingly, this criterion denies the traditional canonical and symmetric expressions of energy-momentum tensor and their associated expressions of angular momentum tensor. The new tensors we propose can be derived as Nöther currents from a Lagrangian with second derivative, and shed new light on the study of nucleon structures.

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I. THREE TYPES OF ENERGY-MOMENTUM TENSOR

Being the conserved currents associated with the symmetries of space-time translation and rotation, energy- momentum and angular momentum tensors are among the most fundamental quantities in both classical and quantum physics, and are the basis for obtaining the structural picture of a physical system, e.g., the quark-gluon structure of nucleon spin, which remains an open problem [1]. There are two popular expressions of energy-momentum tensor, the canonical one and the symmetric one, each has a corresponding expression of angular momentum tensor. In this paper, we present a new type of energy-momentum and angular momentum tensors, and apply them to the study of nucleon structures. To show the difference, let us first give the explicit expression of our new energy-momentum tensor. For a free field ϕ_a , it is

$$T_{\text{new}}^{\mu\nu} = -\frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_\mu \phi_a)} \overleftrightarrow{\partial}^\nu \phi_a, \quad \overleftrightarrow{\partial}^\nu = \frac{1}{2}(\overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu). \quad (1)$$

Here, $\mathcal{L}_{\text{st}}(\phi_a, \partial_\mu \phi_a)$ is the conventional expression of Lagrangian in terms of the field ϕ_a and its first derivative. We take the metric with signature $(-+++)$. In comparison, the canonical expression is

$$T_{\text{cano}}^{\mu\nu} = -\frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a + g^{\mu\nu} \mathcal{L}_{\text{st}}. \quad (2)$$

For example, for the most familiar electromagnetic field with Lagrangian $\mathcal{L}_{\text{st}}^A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, the three types of energy-momentum tensor read:

$$^A T_{\text{new}}^{\mu\nu} = F^{\mu\rho} \overleftrightarrow{\partial}^\nu A_\rho = \frac{1}{2}(F^{\mu\rho} \partial^\nu A_\rho - A_\rho \partial^\nu F^{\mu\rho}), \quad (3a)$$

$$^A T_{\text{cano}}^{\mu\nu} = F^{\mu\rho} \partial^\nu A_\rho + g^{\mu\nu} \mathcal{L}_{\text{st}}, \quad (3b)$$

$$^A T_{\text{symm}}^{\mu\nu} = F^{\mu\rho} F^\nu{}_\rho + g^{\mu\nu} \mathcal{L}_{\text{st}}. \quad (3c)$$

II. JUSTIFICATION OF THE NEW EXPRESSION

Our motivation to re-examine the expressions of energy-momentum and angular momentum tensors is that the conserved currents are not uniquely determined by the conservation laws, which can only prescribe total conserved charges; and much debate arose, especially in the attempt to decomposing the nucleon spin into the spin and orbital contributions of

quarks and gluons (see Refs. [1–3] for most recent discussions). On the other hand, the conserved current *densities* do have independent physical meanings. The most familiar example is that the energy-momentum tensor acts as the source of gravitational field in Einstein’s gravitational theory. The less known spin current, in Einstein-Cartan theory [4], is taken as the source of space-time torsion. In either the Einstein theory or the Einstein-Cartan theory, however, the energy-momentum tensor and the spin tensor are derived *post priori* from the action constructed with minimal coupling. In this paper, we seek to set *a priori* constraint on the energy-momentum and angular momentum tensors.

Our consideration is that *if a quantum wave is in mutual eigen-state of more than one physical observables, and a simultaneous measurement of these observables can be performed, then the currents associated with these observables must be proportional to each other.*

The hint on such correlation of currents comes from classical particles: When one catches a classical particle, one catches all its physical quantities: charge, energy, momentum, etc. Thus, for a beam of identical particles with the same energy ε and momentum p_j for each particle, the energy flux density $\vec{\mathcal{K}}_0$ must be proportional to the momentum flux density $\vec{\mathcal{K}}_j$:

$$\frac{\vec{\mathcal{K}}_0}{\varepsilon} = \frac{\vec{\mathcal{K}}_j}{p_j} = \vec{\mathcal{K}}_n, \quad (4)$$

with $\vec{\mathcal{K}}_n$ the flux density of particle number. [The case will be trivial if all components of p_j are identical for the particles, but non-trivial cases can be designed if just one or two components of p_j are set identical. The same remark applies to the discussion of quantum wave below.]

By the assumption of quantum measurement, when a quantum wave collapses to a local spot, all its physical quantities will localize simultaneously to that same spot. In this way, a quantum wave should exhibit similar correlation of currents as for classical particles: If the wave is in mutual eigen-state of energy ε and momentum p_j , then the density of energy flow T^{i0} and the density of momentum flow T^{ij} must satisfy a constraint similar to Eq. (4):

$$\frac{T^{i0}}{\varepsilon} = \frac{T^{ij}}{p_j}, \quad (5)$$

so that one can have

$$\frac{T^{i0} \cdot dS_i}{\varepsilon} = \frac{T^{ij} \cdot dS_i}{p_j} = \frac{dN}{dt}, \quad (6)$$

where N is the number of particles received at the surface element $d\vec{S}$.

It is interesting and surprising that the conventional expressions of energy-momentum tensor do not show the above correlation. For example, taking the canonical expression in Eq. (2), and making use of the eigen-state assumption $\partial^0 \phi_a = i\varepsilon \phi_a$, $\partial^j \phi_a = ip_j \phi_a$, we have:

$$T_{\text{cano}}^{i0} \rightarrow -i\varepsilon \frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_i \phi_a)} \phi_a, \quad (7a)$$

$$T_{\text{cano}}^{ij} \rightarrow -ip_j \frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_i \phi_a)} \phi_a + \delta_{ij} \mathcal{L}_{\text{st}}. \quad (7b)$$

This satisfies the constraint in Eq. (5) for the transverse momentum flow, namely T^{ij} with $i \neq j$. But for the longitudinal momentum flow T^{jj} , the Lagrangian term in Eq. (7b) makes a trouble for Eq. (5), except for the Dirac field ψ which has $\mathcal{L}_{\text{st}}^\psi = 0$ when applying the equation of motion.

Such a Lagrangian term also exists in the symmetric expression of energy-momentum tensor, which therefore does not fulfill the constraint in Eq. (5), either. In fact, the symmetric energy-momentum tensor stands an even worse situation with respect to such a constraint: One can check with the familiar electromagnetic field that ${}^A T_{\text{symm}}^{\mu\nu}$ in Eq. (3c) does not guarantee Eq. (5) even for $i \neq j$.

Our new expression of energy-momentum tensor in Eq. (1) does not contain the Lagrangian term. In the next section, we derive an equivalent and more illuminating expression than Eq. (1) to display that such current-correlation property can be safely guaranteed for a quantum wave in mutual eigen-state of energy ε and momentum p_j .

III. DERIVATION AS NÖTHER CURRENT

The conventional canonical energy-momentum tensor is derived as a Nöther current with the conventional Lagrangian $\mathcal{L}_{\text{st}}(\phi_a, \partial_\mu \phi_a)$. From our above discussion, we see that it almost satisfies the constraint in Eq. (5), except for the Lagrangian term which does not in general vanish. Since the Lagrangian of a field can be modified by a surface term without changing the equation of motion, this gives us a hint that if we can find a general expression of Lagrangian which always vanishes after applying the equation of motion, then the derived Nöther current will automatically satisfy the constraint in Eq. (5). In what follows, we show that it is indeed so. We will first concentrate on the free-field case which is already non-trivial, and discuss the interacting case in section VI, in connection with the quark-gluon structure of nucleon.

The conventional standard Lagrangians of free scalar, Dirac, and vector fields take the following forms, respectively:

$$\mathcal{L}_{\text{st}}^\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2, \quad (8a)$$

$$\mathcal{L}_{\text{st}}^\psi = \frac{1}{2}\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + h.c., \quad (8b)$$

$$\mathcal{L}_{\text{st}}^A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (8c)$$

By noticing that a free-field Lagrangian $\mathcal{L}_{\text{st}}(\phi_a, \partial_\mu\phi_a)$ is necessarily quadratic in the field variable and its derivative, it can be put in a unified form:

$$\mathcal{L}_{\text{st}}(\phi_a, \partial_\mu\phi_a) = \frac{1}{2}\left[\phi_a\frac{\partial\mathcal{L}_{\text{st}}}{\partial\phi_a} + (\partial_\mu\phi_a)\frac{\partial\mathcal{L}_{\text{st}}}{\partial(\partial_\mu\phi_a)}\right]. \quad (9)$$

By adding a proper surface term, we obtain the desired new expression of Lagrangian $\mathcal{L}_{\text{new}}(\phi_a, \partial_\mu\phi_a, \partial_\mu\partial_\nu\phi_a)$:

$$\mathcal{L}_{\text{new}} = \mathcal{L}_{\text{st}} - \frac{1}{2}\partial_\mu\left[\phi_a\frac{\partial\mathcal{L}_{\text{st}}}{\partial(\partial_\mu\phi_a)}\right] \quad (10a)$$

$$= \frac{1}{2}\phi_a\left[\frac{\partial\mathcal{L}_{\text{st}}}{\partial\phi_a} - \partial_\mu\frac{\partial\mathcal{L}_{\text{st}}}{\partial(\partial_\mu\phi_a)}\right], \quad (10b)$$

which clearly vanishes by the Euler-Lagrange equation of motion.

The explicit forms of our new Lagrangian for the scalar, Dirac, and vector fields are:

$$\mathcal{L}_{\text{new}}^\phi = \frac{1}{2}\phi(\partial_\mu\partial^\mu - m^2)\phi, \quad (11a)$$

$$\mathcal{L}_{\text{new}}^\psi = \frac{1}{2}\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + h.c., \quad (11b)$$

$$\mathcal{L}_{\text{new}}^A = \frac{1}{2}A_\nu\partial_\mu F^{\mu\nu}. \quad (11c)$$

For the Dirac field, the “new” Lagrangian actually equals the traditional expression, which is already zero by the equation of motion.

Notice that the new Lagrangian \mathcal{L}_{new} contains a second derivative, thus the derivation of Nöther current is a little bit (but not much) more involved [5]. The result is:

$$T_{\text{new}}^{\mu\nu} = -i\left[\frac{\partial\mathcal{L}_{\text{new}}}{\partial(\partial_\mu\phi_a)} + \frac{\partial\mathcal{L}_{\text{new}}}{\partial(\partial_\mu\partial_\sigma\phi_a)}\partial_\sigma - \partial_\sigma\frac{\partial\mathcal{L}_{\text{new}}}{\partial(\partial_\sigma\partial_\mu\phi_a)}\right]\mathbf{P}^\nu\phi_a, \quad (12)$$

where $\mathbf{P}^\nu = -i\partial^\nu$ is the quantum-mechanical four-momentum operator. We call this a “hyper-canonical” form, as it is a single expression with the single operator inserted for the desired observable. Such a hyper-canonical form clearly guarantees the current-correlation

property as we elaborated above for a quantum wave in mutual eigen-state of two or more components of \mathbf{P}^ν .

By a slight algebra, Eq. (12) can be converted into the more convenient expression in Eq. (1), expressed with the conventional Lagrangian \mathcal{L}_{st} containing only the first derivative.

As a cross-check, the difference between $T_{\text{new}}^{\mu\nu}$ and $T_{\text{cano}}^{\mu\nu}$ is a total-divergence term:

$$T_{\text{new}}^{\mu\nu} = T_{\text{cano}}^{\mu\nu} + \partial_\lambda \mathcal{K}^{[\lambda\mu]\nu}, \quad (13a)$$

$$\mathcal{K}^{[\lambda\mu]\nu} = \frac{1}{2} \left(g^{\lambda\nu} \frac{\partial \mathcal{L}_{\text{st}}}{\partial (\partial_\mu \phi_a)} - g^{\mu\nu} \frac{\partial \mathcal{L}_{\text{st}}}{\partial (\partial_\lambda \phi_a)} \right) \phi_a. \quad (13b)$$

Here, $\mathcal{K}^{[\lambda\mu]\nu}$ is antisymmetric in its first two indices. As a consequence, $T_{\text{new}}^{\mu\nu}$ satisfies the same conservation law and gives the same conserved four-momentum as does by $T_{\text{cano}}^{\mu\nu}$:

$$\partial_\mu T_{\text{cano}}^{\mu\nu} = \partial_\mu T_{\text{new}}^{\mu\nu} = 0, \quad (14a)$$

$$P^\nu = \int d^3x T_{\text{cano}}^{0\nu} = \int d^3x T_{\text{new}}^{0\nu}. \quad (14b)$$

IV. THE ANGULAR MOMENTUM TENSOR

Due to the vanishing of the new Lagrangian \mathcal{L}_{new} under the equation of motion, the “hyper-canonical” structure in Eq. (12) actually applies to any Nöther current. The more non-trivial example is the new angular momentum tensor:

$$M_{\text{new}}^{\lambda\mu\nu} = -i \left[\frac{\partial \mathcal{L}_{\text{new}}}{\partial (\partial_\lambda \phi_a)} + \frac{\partial \mathcal{L}_{\text{new}}}{\partial (\partial_\lambda \partial_\sigma \phi_a)} \partial_\sigma - \partial_\sigma \frac{\partial \mathcal{L}_{\text{new}}}{\partial (\partial_\sigma \partial_\lambda \phi_a)} \right] \mathbf{J}_{ab}^{\mu\nu} \phi_b. \quad (15)$$

Here, $\mathbf{J}_{ab}^{\mu\nu}$ is the total angular momentum operator that governs the transformation of ϕ_a under an infinitesimal Lorentz transformation $x_\mu \rightarrow x'_\mu = x_\mu + \omega_{\mu\nu} x^\nu$:

$$\delta \phi_a(x) = \frac{1}{2} \omega_{\mu\nu} [(x^\mu \partial^\nu - x^\nu \partial^\mu) \delta_{ab} + i \mathbf{S}_{ab}^{\mu\nu}] \phi_b(x) \equiv \frac{1}{2} \omega_{\mu\nu} i \mathbf{J}_{ab}^{\mu\nu} \phi_b. \quad (16)$$

Namely, $\mathbf{J}^{\mu\nu} = (x^\mu \mathbf{P}^\nu - x^\nu \mathbf{P}^\mu) + \mathbf{S}^{\mu\nu} = \mathbf{L}^{\mu\nu} + \mathbf{S}^{\mu\nu}$. It relates to the usual angular momentum operator by $\mathbf{J}_i = \frac{1}{2} \epsilon_{ijk} \mathbf{J}^{jk}$. The same hyper-canonical structure in Eqs. (12) and (15) guarantees that if the wave ϕ_a is a mutual eigen-state of energy and angular momentum, say, $\mathbf{J}_{ab}^{12} \phi_b = j_3 \phi_a$, then the fluxes of energy and angular momentum will be correlated:

$$\frac{T^{i0}}{\varepsilon} = \frac{M^{i12}}{j_3}. \quad (17)$$

In comparison, the canonical angular momentum tensor,

$$M_{\text{cano}}^{\lambda\mu\nu} = x^\mu T_{\text{cano}}^{\lambda\nu} - x^\nu T_{\text{cano}}^{\lambda\mu} - i \frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_\lambda \phi_a)} \mathbf{S}_{ab}^{\mu\nu} \phi_b, \quad (18)$$

cannot be put in a form in which the angular momentum operator $\mathbf{J}_{ab}^{\mu\nu}$ appears as a whole, and thus does not satisfy the quantum constraint as we put above. [Specifically, $M_{\text{cano}}^{\lambda\mu\nu}$ contradicts Eq. (17) for the transverse angular momentum flux with $i = 1$ or 2 ; only the longitudinal flux M_{cano}^{312} satisfies Eq. (17).] It is easy to check that the angular momentum tensor associated with the symmetric energy-momentum tensor,

$$M_{\text{symm}}^{\lambda\mu\nu} = x^\mu T_{\text{symm}}^{\lambda\nu} - x^\nu T_{\text{symm}}^{\lambda\mu}, \quad (19)$$

stands a even worse situation with respect to Eq. (17).

To make a more detailed comparison with $M_{\text{cano}}^{\lambda\mu\nu}$, we convert $M_{\text{new}}^{\lambda\mu\nu}$ in Eq. (15) into a more conventional form:

$$M_{\text{new}}^{\lambda\mu\nu} = x^\mu T_{\text{new}}^{\lambda\nu} - x^\nu T_{\text{new}}^{\lambda\mu} - i \frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_\lambda \phi_a)} \mathbf{S}_{ab}^{\mu\nu} \phi_b + \frac{1}{2} \left[g^{\lambda\mu} \frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_\nu \phi_a)} - g^{\lambda\nu} \frac{\partial \mathcal{L}_{\text{st}}}{\partial(\partial_\mu \phi_a)} \right] \phi_a. \quad (20)$$

By this form, it can be checked that the difference between $M_{\text{new}}^{\lambda\mu\nu}$ and $M_{\text{cano}}^{\lambda\mu\nu}$ is again a total-divergence term:

$$M_{\text{new}}^{\lambda\mu\nu} = M_{\text{cano}}^{\lambda\mu\nu} + \partial_\rho (x^\mu \mathcal{K}^{[\rho\lambda]\nu} - x^\nu \mathcal{K}^{[\rho\lambda]\mu}), \quad (21)$$

with the same $\mathcal{K}^{[\rho\lambda]\nu}$ as in Eq. (13b). Therefore, $M_{\text{new}}^{\lambda\mu\nu}$ and $M_{\text{cano}}^{\lambda\mu\nu}$ satisfy the same conservation law and give the same conserved angular momentum:

$$\partial_\lambda M_{\text{cano}}^{\lambda\mu\nu} = \partial_\lambda M_{\text{new}}^{\lambda\mu\nu} = 0, \quad (22a)$$

$$J^{\mu\nu} = \int d^3x M_{\text{cano}}^{0\mu\nu} = \int d^3x M_{\text{new}}^{0\mu\nu}. \quad (22b)$$

Comparing $M_{\text{new}}^{\lambda\mu\nu}$ in Eq. (20) with $M_{\text{cano}}^{\lambda\mu\nu}$ in Eq. (18), we see that the last term in Eq. (20) can be regarded as an extra spin current. Certainly, the integrated spin “charge” is not altered by this extra current, which does not contribute to the component M_{new}^{0ij} .

V. EXPLICIT EXPRESSIONS

For the convenience of future references, we summarize here the explicit expressions. For the scalar field, we have

$$\phi T_{\text{new}}^{\mu\nu} = \partial^\mu \phi \overleftrightarrow{\partial}^\nu \phi = \frac{1}{2}(\partial^\mu \phi \partial^\nu \phi - \phi \partial^\mu \partial^\nu \phi), \quad (23a)$$

$$\phi M_{\text{new}}^{\lambda\mu\nu} = \phi T^{\lambda\nu} x_{\text{new}}^\mu - \phi T_{\text{new}}^{\lambda\mu} x^\nu + \frac{1}{2}\phi(g^{\lambda\nu} \partial^\mu \phi - g^{\lambda\mu} \partial^\nu \phi). \quad (23b)$$

Note that $\phi T_{\text{new}}^{\mu\nu}$ is still symmetric, yet it is different from the conventional symmetric expression of energy-momentum tensor, which for a scalar field coincides with the canonical expression. More remarkably, the scalar field does acquire the extra spin current, though its spin charge is still zero.

For the Dirac field, the conventional choice of Lagrangian is already zero by the equation of motion, therefore our “new” expressions coincide with the conventional canonical expressions:

$$\psi T_{\text{new}}^{\mu\nu} = \psi T_{\text{cano}}^{\mu\nu} = -i\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi = \frac{-i}{2}[\bar{\psi}\gamma^\mu \partial^\nu \psi - (\partial^\nu \bar{\psi})\gamma^\mu \psi], \quad (24a)$$

$$\psi M_{\text{new}}^{\lambda\mu\nu} = \psi M_{\text{cano}}^{\lambda\mu\nu} = \psi T_{\text{new}}^{\lambda\nu} x^\mu - \psi T_{\text{new}}^{\lambda\mu} x^\nu + \frac{1}{2}\varepsilon^{\lambda\mu\nu\rho}\bar{\psi}\gamma_\rho\gamma_5\psi. \quad (24b)$$

So, for the Dirac field, all we add by our current-correlation analysis is that the symmetric expression of energy-momentum tensor is disfavored.

For the electromagnetic field, we have already put the explicit expression of our new energy-momentum tensor in Eq. (3a), and the new angular momentum tensor is

$$^A M_{\text{new}}^{\lambda\mu\nu} = ^A T_{\text{new}}^{\lambda\nu} x^\mu - ^A T_{\text{new}}^{\lambda\mu} x^\nu + A^\mu F^{\nu\lambda} - A^\nu F^{\mu\lambda} + \frac{1}{2}(g^{\lambda\nu} A_\rho F^{\mu\rho} - g^{\lambda\mu} A_\rho F^{\nu\rho}). \quad (25)$$

It contains both the “traditional” spin current and the extra spin current, but only the former contributes the spin charge.

VI. THE QUARK-GLUON SYSTEM AND NUCLEON STRUCTURE

For the interacting fields, the Lagrangian necessarily contains higher-than-quadratic terms, therefore, the expression in Eq. (9) does not hold, and we cannot reach the hypercanonical form as in Eqs. (12) and (15) for the free fields. This is actually reasonable, because generally the individual field of an interacting system can no longer be separately

in the eigenstate of energy, momentum, or angular momentum, so our current-correlation analysis in quantum measurement doe not apply. All the guidance we have here is that the Nöther currents should reduce to the hyper-canonical expressions for the free fields when the coupling constant goes to zero, while in the presence of interaction, they must satisfy the same conservation laws and give the same conserved charges as the conventional expressions do. Let us tentatively modify an interacting Lagrangian with the same surface term as in Eq. (10a). [Certainly, in the presence of interaction we can no longer reach Eq. (10b).] For example, take the standard expression of QCD Lagrangian,

$$\begin{aligned}\mathcal{L}_{\text{QCD}} &= \bar{\psi}(i\gamma^\mu \overleftrightarrow{\partial}_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + g\bar{\psi}\gamma^\mu t^a \psi A_\mu^a \\ &\equiv \mathcal{L}_q + \mathcal{L}_g + \mathcal{L}_{\text{qg}},\end{aligned}\tag{26}$$

we get

$$\widetilde{\mathcal{L}}_{\text{QCD}} \equiv \mathcal{L}_{\text{QCD}} - \frac{1}{2}\partial_\mu\left(\frac{\partial\mathcal{L}_{\text{QCD}}}{\partial(\partial_\mu\phi_a)}\phi_a\right) = \mathcal{L}_q + \widetilde{\mathcal{L}}_g + \mathcal{L}_{\text{qg}},\tag{27}$$

where the modified gluon part is

$$\widetilde{\mathcal{L}}_g = \frac{1}{2}A_\nu^a\partial_\mu F^{\mu\nu a} - \frac{1}{4}gf^{abc}F^{\mu\nu a}A_\mu^b A_\nu^c.\tag{28}$$

With this Lagrangian, the new energy-momentum and angular momentum tensors are derived to be

$${}^{\text{QCD}}\mathcal{T}_{\text{new}}^{\mu\nu} = -i\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi + F^{\mu\rho a}\overleftrightarrow{\partial}^\nu A_\rho^a + g^{\mu\nu}\widetilde{\mathcal{L}}_{\text{QCD}},\tag{29a}$$

$$\begin{aligned}{}^{\text{QCD}}\mathcal{M}_{\text{new}}^{\lambda\mu\nu} &= -i\bar{\psi}\gamma^\lambda x^{[\mu}\overleftrightarrow{\partial}^{\nu]}\psi \longrightarrow \mathcal{M}_{\text{q,orbital}}^{\lambda\mu\nu} \\ &\quad + F^{\lambda\rho a}x^{[\mu}\overleftrightarrow{\partial}^{\nu]}A_\rho^a \longrightarrow \mathcal{M}_{\text{g,orbital}}^{\lambda\mu\nu} \\ &\quad + \frac{1}{2}\varepsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma^5\psi \longrightarrow \mathcal{M}_{\text{q,spin}}^{\lambda\mu\nu} \\ &\quad + F_a^{\lambda[\mu}A_a^{\nu]} + \frac{1}{2}g^{\lambda[\nu}F^{\mu]\rho a}A_\rho^a \longrightarrow \mathcal{M}_{\text{g,spin}}^{\lambda\mu\nu} \\ &\quad + x^{[\mu}g^{\nu]\lambda}\widetilde{\mathcal{L}}_{\text{QCD}} \longrightarrow \mathcal{M}_{\text{boost}}^{\lambda\mu\nu},\end{aligned}\tag{29b}$$

where the indices in square brackets are to be anti-symmetrized. They can indeed reduce to the free-field expressions in the absence of interaction. Moreover, for the momentum density, the Lagrangian term in ${}^{\text{QCD}}\mathcal{T}_{\text{new}}^{0i}$ drops out, and for the angular momentum density, the boost term in ${}^{\text{QCD}}\mathcal{M}_{\text{new}}^{0ij}$ drops out, so we get exactly the same expression as if quark and gluon exist separately.

Like the canonical expressions in gauge theory, our new expressions of energy-momentum tensor and angular momentum tensor are naively gauge-dependent. To achieve gauge-invariance, one can employ the method as discussed in Refs. [1, 6, 7], by separating the gauge field into a physical part and a pure-gauge part:

$$A^\mu = A_{pure}^\mu + A_{phys}^\mu. \quad (30)$$

With this method, Eqs. (29) can be upgraded to be gauge-invariant:

$${}^{\text{QCD}}T_{\text{new}}^{\mu\nu} = -i\bar{\psi}\gamma^\mu \overleftrightarrow{D}_{pure}^\nu \psi + 2\text{Tr}[F^{\mu\rho} \overleftrightarrow{\mathcal{D}}_{pure}^\nu A_\rho^{phys}] + g^{\mu\nu} \widetilde{\mathcal{L}}_{\text{QCD}}^{phys}, \quad (31a)$$

$$\begin{aligned} {}^{\text{QCD}}M_{\text{new}}^{\mu\nu\lambda} = & -i\bar{\psi}\gamma^\lambda x^{[\mu} \overleftrightarrow{D}_{pure}^{\nu]} \psi \longrightarrow M_{\text{q,orbital}}^{\lambda\mu\nu} \\ & + 2\text{Tr}[F^{\lambda\rho} x^{[\mu} \overleftrightarrow{\mathcal{D}}_{pure}^{\nu]} A_\rho^{phys}] \longrightarrow M_{\text{g,orbital}}^{\lambda\mu\nu} \\ & + \frac{1}{2}\varepsilon^{\mu\nu\lambda\sigma} \bar{\psi}\gamma_\sigma \gamma^5 \psi \longrightarrow M_{\text{q,spin}}^{\lambda\mu\nu} \\ & + 2\text{Tr}[F^{\lambda[\mu} A_{phys}^{\nu]} + \frac{1}{2}g^{\lambda[\nu} F^{\mu]\rho} A_\rho^{phys}] \longrightarrow M_{\text{g,spin}}^{\lambda\mu\nu} \\ & + x^{[\mu} g^{\nu]\lambda} \widetilde{\mathcal{L}}_{\text{QCD}}^{phys} \longrightarrow M_{\text{boost}}^{\lambda\mu\nu}, \end{aligned} \quad (31b)$$

where the gauge-covariant derivative is constructed with the pure-gauge field A_{pure}^μ , and the gauge-invariant modified Lagrangian is

$$\widetilde{\mathcal{L}}_{\text{QCD}}^{phys} = \mathcal{L}_{\text{QCD}} - \frac{1}{2}\partial_\mu \left(\frac{\partial \mathcal{L}_{\text{QCD}}}{\partial(\partial_\mu A_\rho)} A_\rho^{phys} \right) = \mathcal{L}_{\text{QCD}} + \partial_\mu \text{Tr}[F^{\mu\rho} A_\rho^{phys}]. \quad (32)$$

If we consider the integrated spin, momentum, or orbital angular momentum of quarks and gluons, each term in Eqs. (31) coincide with the corresponding term in Ref. [7], this justified the gauge-invariant canonical decomposition of nucleon momentum and spin in Ref. [7]. Note, however, that if the flux densities of momentum and angular momentum are considered, Eqs. (29) and (31) differ from the canonical expressions.

VII. DISCUSSION

In this paper, we employed the particle-wave duality in quantum mechanics to set a first-principle constraint on the expressions of energy-momentum tensor and angular momentum tensor. It should be reminded, nevertheless, that although the foundation of our whole discussion — the mutual conservation of multiple physical quantities during a quantum measurement, or the simultaneous localization of multiple quantities to the same spot during

the collapse of a quantum wave — sounds very natural and reasonable, it yet has never been actually verified by experiment, as far as we know. We encourage that such a fundamental, important, and taken-for-granted quantum property be tested.

It is interesting to note that in Ref. [8, 9], the dual symmetry between electric field \vec{E} and magnetic field \vec{B} is employed to derive alternative expressions for the energy-momentum and angular momentum tensors of the electromagnetic field. We checked that their expressions also satisfy the current correlation as we discussed here. Certainly, the method based on \vec{E} - \vec{B} dual-symmetry cannot be applied to a general field.

The most important use of energy-momentum tensor is to generate gravity. In another paper [10], we will construct a theory with our new energy-momentum tensor and spin tensor acting as the source of gravity and torsion.

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